

Maths for Computing

Tutorial 11

1. Prove that every connected graph of n vertices contains at least $n - 1$ edges.
2. Let $n \geq 2$ be an integer, and let $a_1 \geq a_2 \geq \dots \geq a_n$ be a sequence of positive integers satisfying $a_1 + a_2 + \dots + a_n = 2n - 2$. Prove that there exists a tree T on n vertices so that the ordered degree sequence of T is a_1, a_2, \dots, a_n .
3. Prove that every graph $G = (V, E)$ with $|V| \geq 3$ and all vertices of degree ≥ 2 contains a cycle of length 3.
4. At most how many edges a graph of n vertices can have so that it does not have an odd length cycle?
5. A round robin football tournament has $2n$ participating teams. Two rounds have been played so far. Prove that we can still split the teams into two groups of n teams each so that no teams of the same group have played each other yet.
6. Prove that every tree has at most one perfect matching. What is the chromatic number of a tree?
7. Is there a bipartite graph with ordered degree sequence $3, 3, 3, 3, 3, 5, 6, 6, 6$?